Symmetry-resolved modular correlation functions in free fermionic theories

Giuseppe Di Giulio

Julius-Maximilian-Universität Würzburg

Talk based on *JHEP* 07 (2023) 058, in collaboration with Johanna Erdmenger

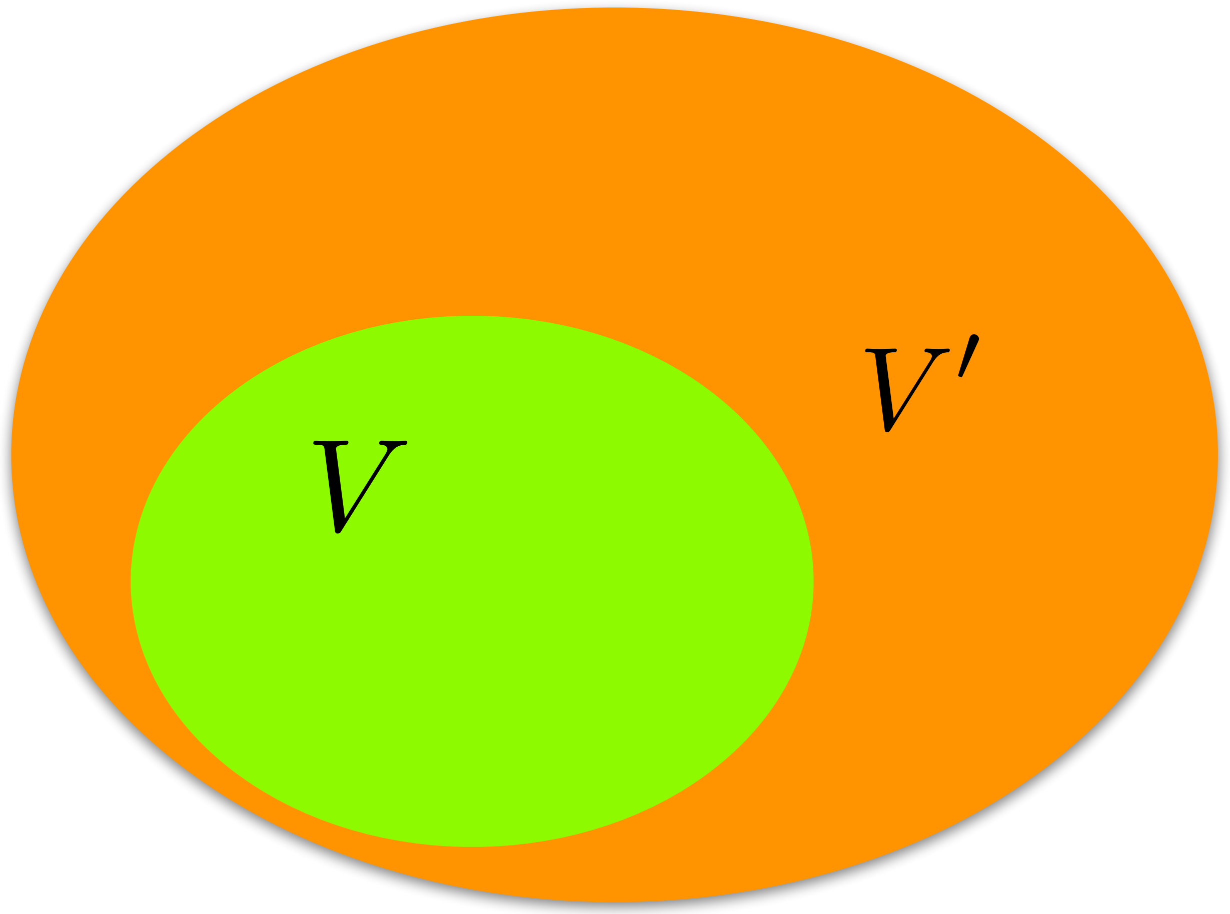
Workshop "Quantum Information, Quantum Matter and Quantum Gravity",
Yukawa Institute for Theoretical Physics, Kyoto University, 3 October 2023

Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

ct.qmat
Complexity and Topology
in Quantum Matter

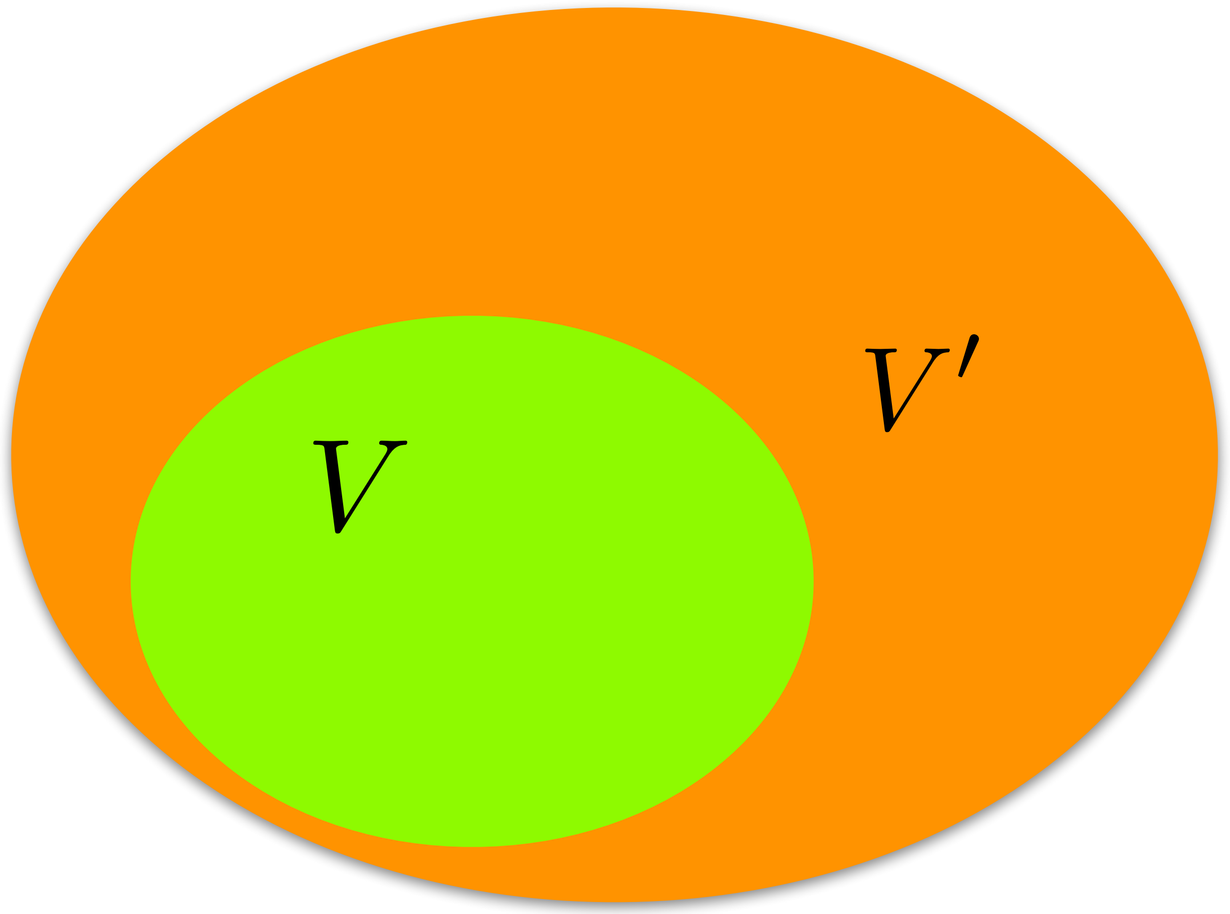
Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $|\psi\rangle \in \mathcal{H}$



Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $\quad |\psi\rangle \in \mathcal{H}$



density matrix $|\psi\rangle\langle\psi|$

$\rho_V = \mathrm{Tr}_{V'}|\psi\rangle\langle\psi|$

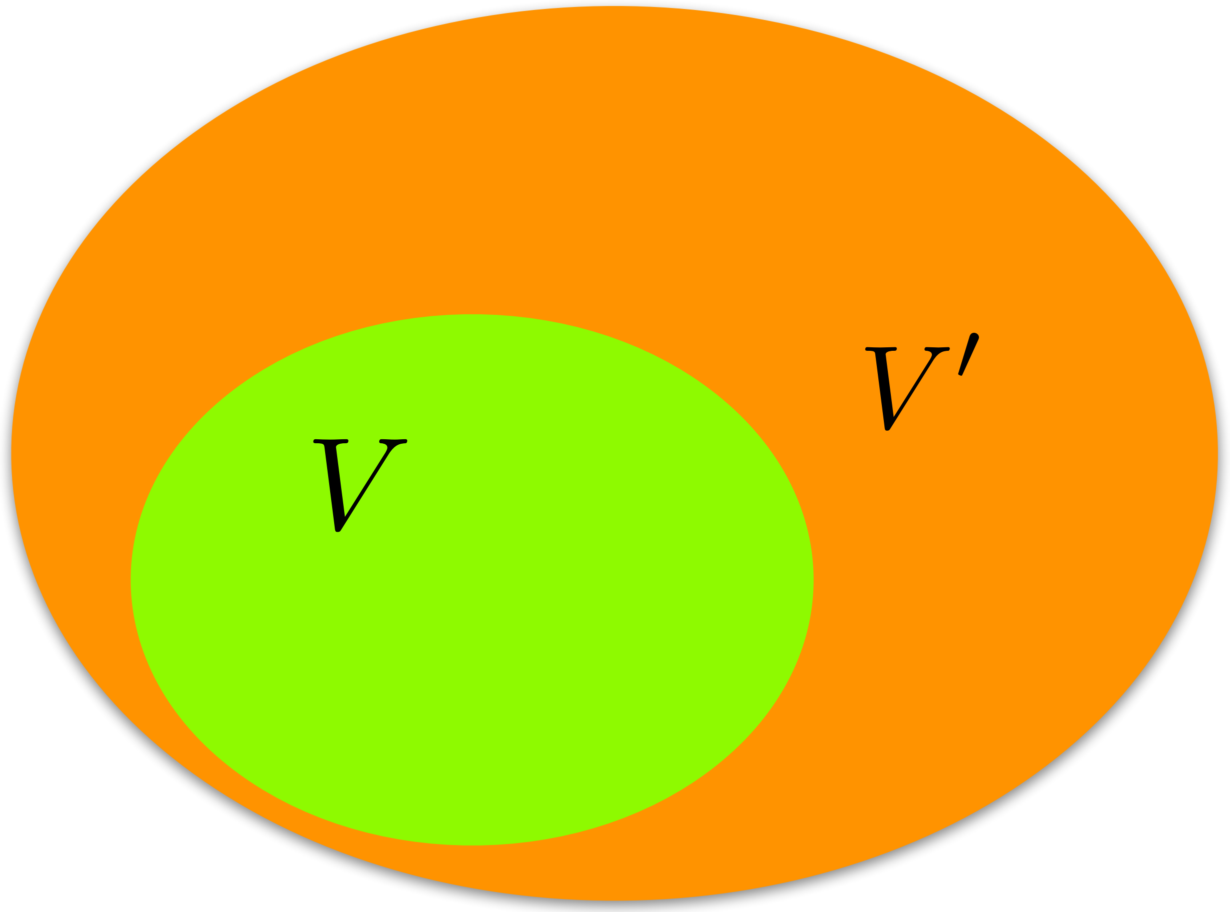
reduced density matrix

$S_V = -\mathrm{Tr}(\rho_V \ln \rho_V)$

entanglement entropy

Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $\quad |\psi\rangle \in \mathcal{H}$



density matrix $|\psi\rangle\langle\psi|$

$$\rho_V = \mathrm{Tr}_{V'} |\psi\rangle\langle\psi|$$

reduced density matrix

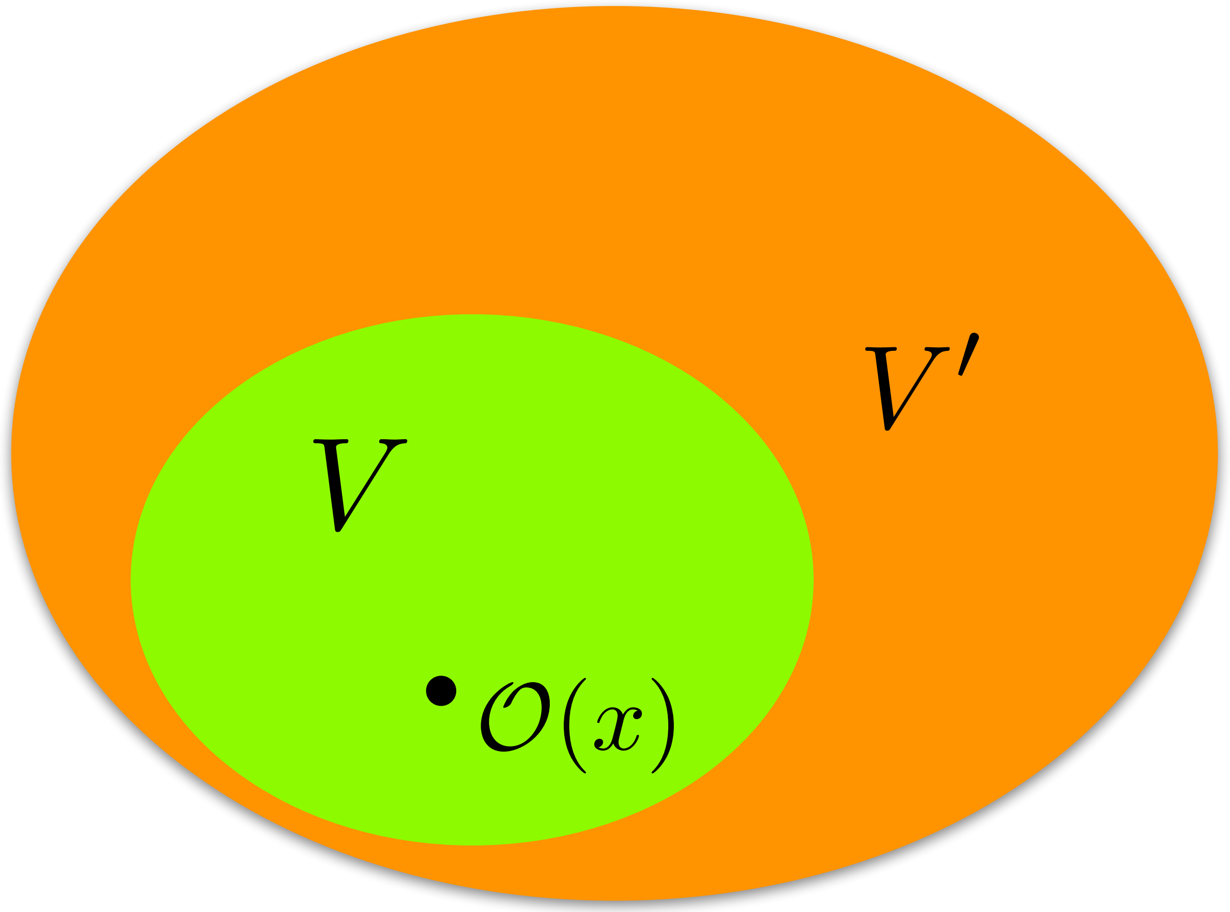
$$S_V = -\mathrm{Tr}(\rho_V \ln \rho_V)$$

entanglement entropy

divergent in QFT, regulated by UV cutoff

Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $\quad |\psi\rangle \in \mathcal{H}$



density matrix $|\psi\rangle\langle\psi|$

$\rho_V = \mathrm{Tr}_{V'} |\psi\rangle\langle\psi|$

reduced density matrix

$S_V = -\mathrm{Tr}(\rho_V \ln \rho_V)$

entanglement entropy

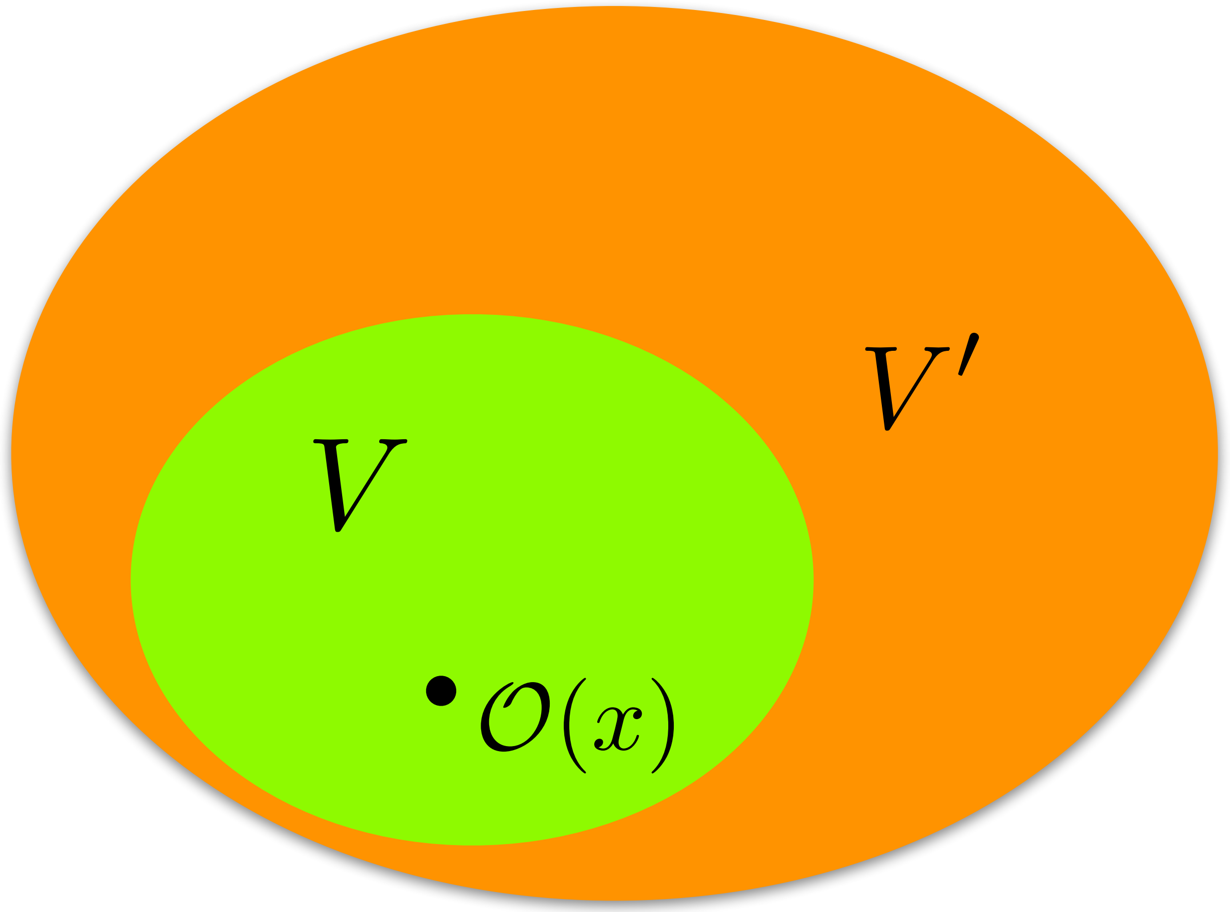
divergent in QFT,
regulated by UV cutoff

Modular flow

$\rho_V^{\mathrm{i}t} \mathcal{O}(x) \rho_V^{-\mathrm{i}t}$

Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $\quad |\psi\rangle \in \mathcal{H}$



density matrix $|\psi\rangle\langle\psi|$

$\rho_V = \mathrm{Tr}_{V'}|\psi\rangle\langle\psi|$

reduced density matrix

$S_V = -\mathrm{Tr}(\rho_V \ln \rho_V)$

entanglement entropy

→ divergent in QFT,
regulated by UV cutoff

Modular flow

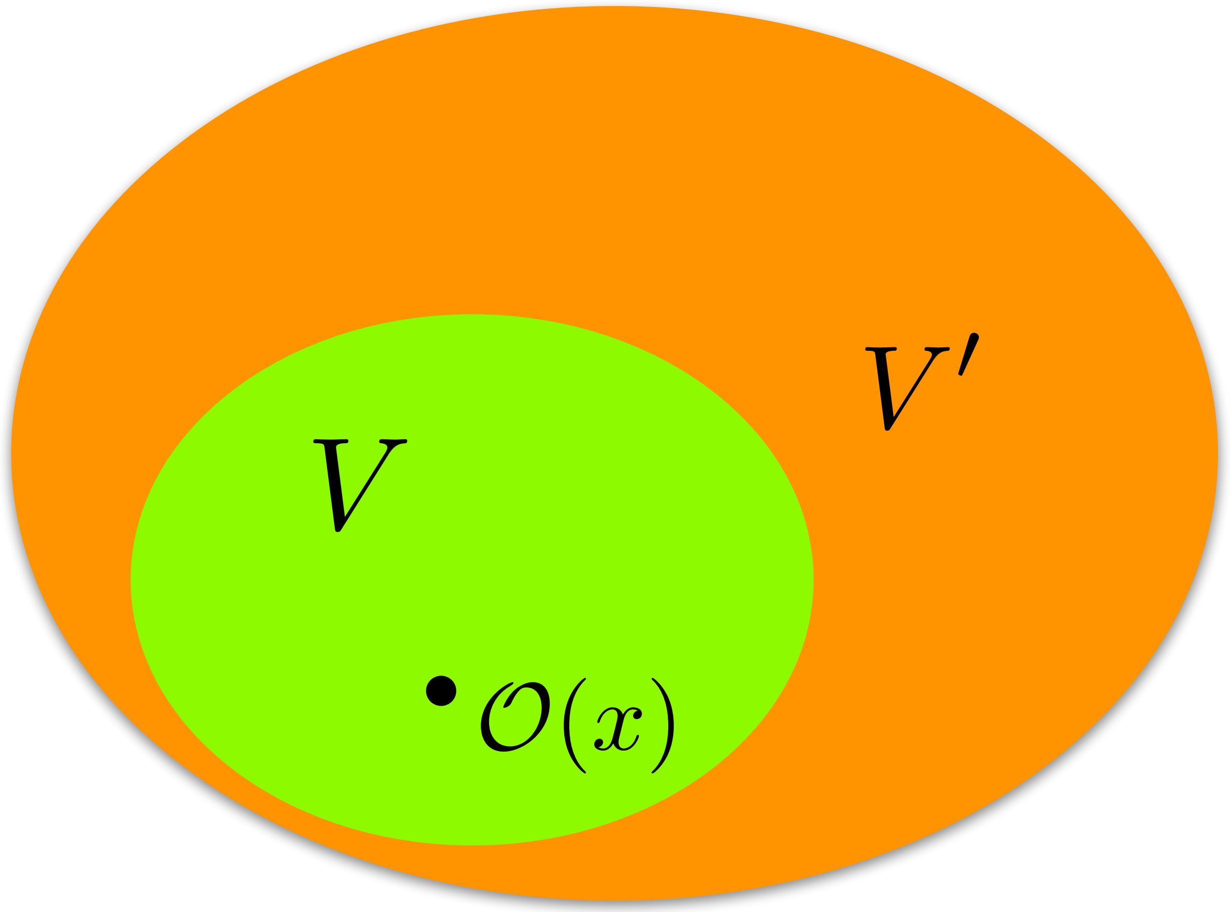
$\rho_V^{it}\mathcal{O}(x)\rho_V^{-it}$

Introduced in algebraic quantum field theory

- rigorously defined in QFT
- involves QFT generalization of reduced density matrices

Entanglement and modular flow in many-body systems

Spatial bipartition into $V \cup V'$ such that $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ $|\psi\rangle \in \mathcal{H}$



density matrix $|\psi\rangle\langle\psi|$

$\rho_V = \mathrm{Tr}_{V'}|\psi\rangle\langle\psi|$

reduced density matrix

$S_V = -\mathrm{Tr}(\rho_V \ln \rho_V)$

entanglement entropy

divergent in QFT, regulated by UV cutoff

Modular flow

$\rho_V^{\mathrm{i}t}\mathcal{O}(x)\rho_V^{-\mathrm{i}t}$

Introduced in algebraic quantum field theory

- rigorously defined in QFT
- involves QFT generalization of reduced density matrices

Application in the **AdS/CFT correspondence**, within the bulk reconstruction program

[Jafferis, Lewkowycz, Maldacena, Suh, ’16; Faulkner, Lewkowycz, ’17; Chandrasekaran, Faulkner, Levine, ’22; Chandrasekaran, Levine, ’22]

Symmetry resolution of the entanglement

New ingredient for analyzing the fine structure of entanglement in theories endowed with a global symmetry

System with a conserved $U(1)$ charge $Q = Q_V + Q_{V'}$

$|\psi\rangle \in \mathcal{H}$ eigenvector of Q

Symmetry resolution of the entanglement

New ingredient for analyzing the fine structure of entanglement in theories endowed with a global symmetry

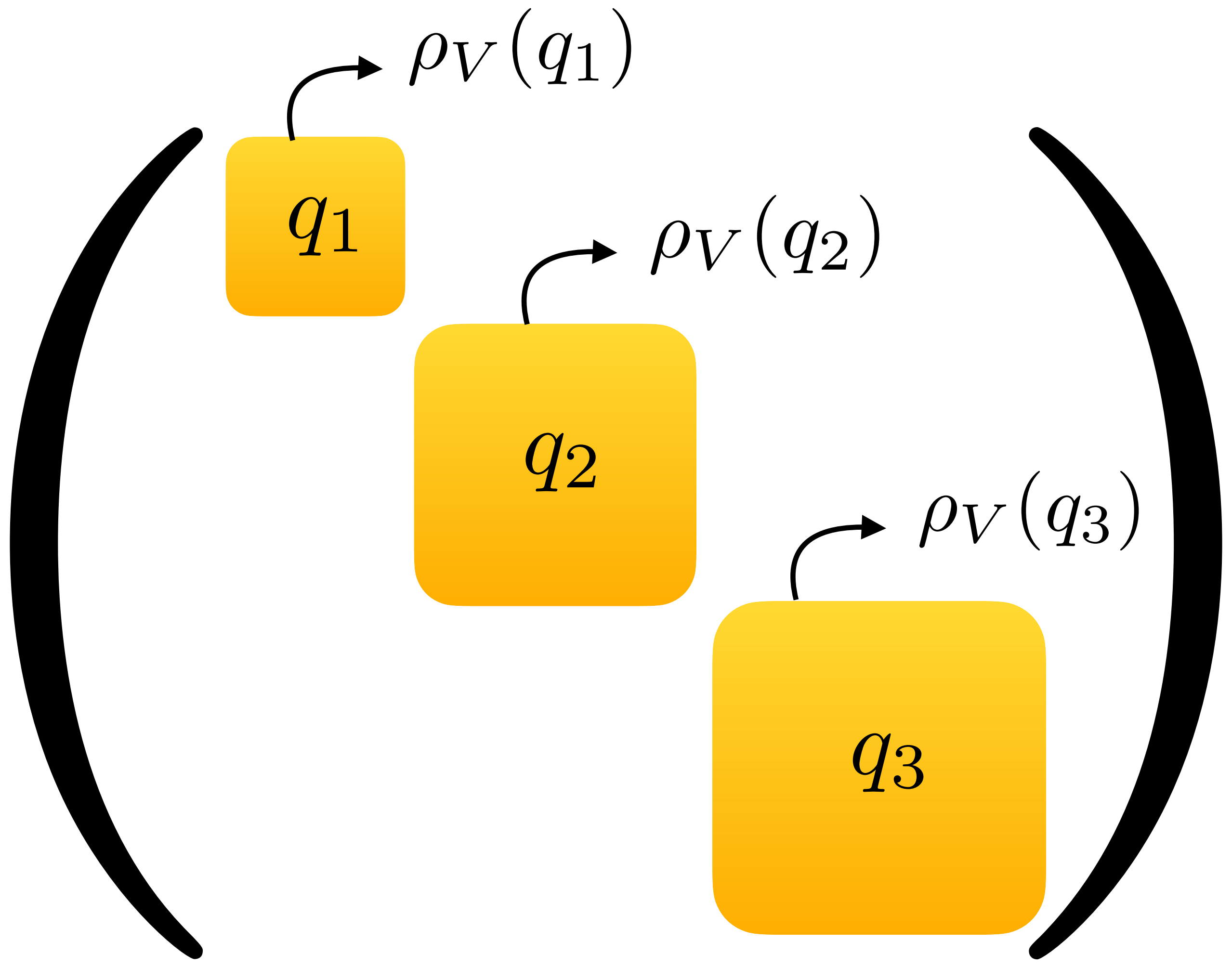
System with a conserved $U(1)$ charge $Q = Q_V + Q_{V'}$

$|\psi\rangle \in \mathcal{H}$ eigenvector of Q

$$[Q_V, \rho_V] = 0$$

$$\rho_V(q) \equiv \frac{\Pi_V(q)\rho_V\Pi_V(q)}{\mathrm{Tr}(\Pi_V(q)\rho_V)} \longrightarrow p_V(q)$$

$$\rho_V =$$



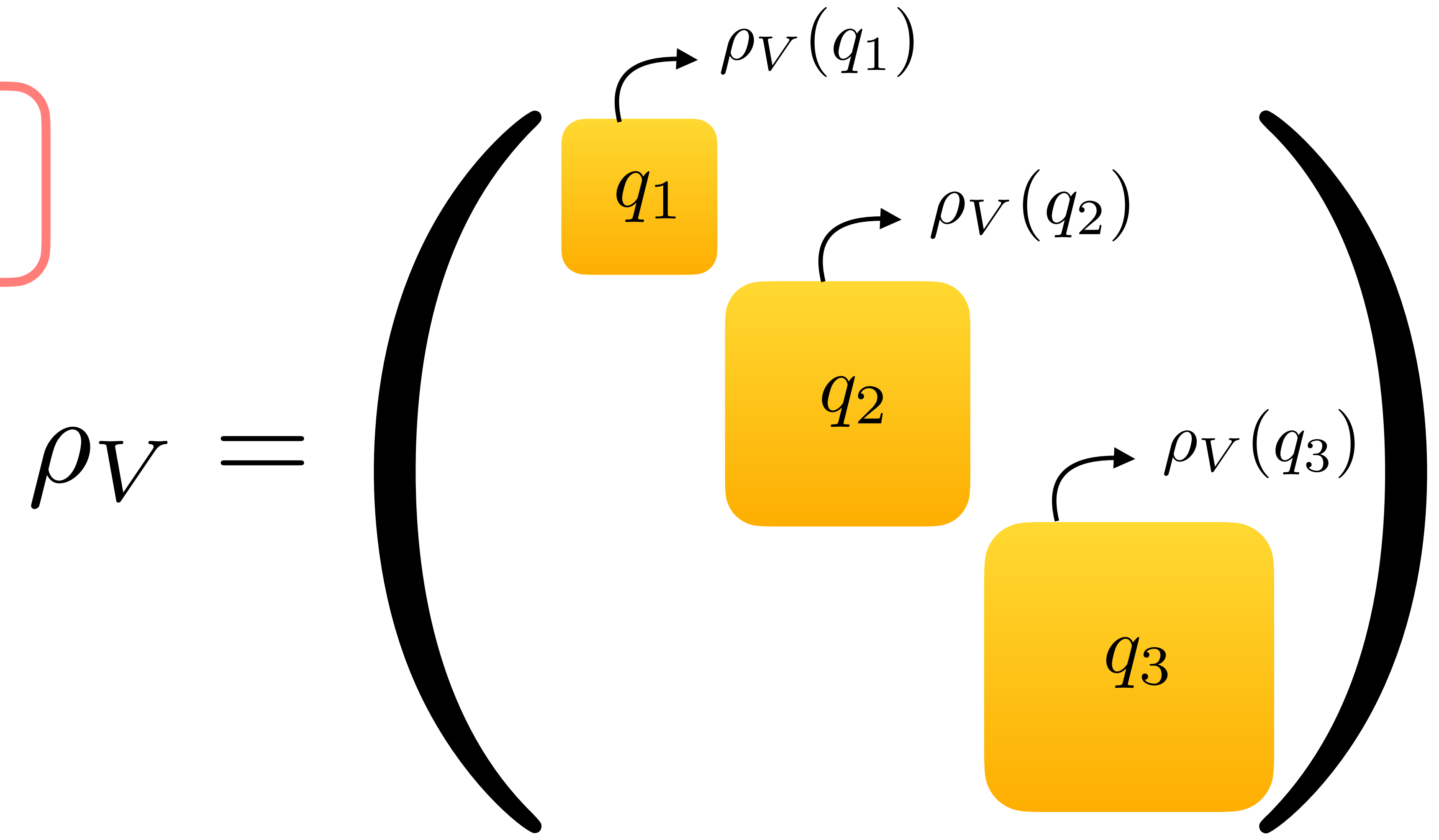
Symmetry resolution of the entanglement

New ingredient for analyzing the fine structure of entanglement in theories endowed with a global symmetry

System with a conserved $U(1)$ charge $Q = Q_V + Q_{V'}$

$|\psi\rangle \in \mathcal{H}$ eigenvector of Q

$$[Q_V, \rho_V] = 0$$



$$\rho_V(q) \equiv \frac{\Pi_V(q)\rho_V\Pi_V(q)}{\mathrm{Tr}(\Pi_V(q)\rho_V)} \longrightarrow p_V(q)$$

Symmetry-resolved entanglement entropy [Laflorencie, Rachel, '14, Sela, Goldstein,'17]

$$S_V(q) = -\mathrm{Tr}\rho_V(q)\ln\rho_V(q) \longrightarrow S_V = \sum_q p_V(q)S_V(q) - \sum_q p_V(q)\ln p_V(q)$$

Charge sector decomposition of entanglement entropy

Symmetry resolution of the entanglement

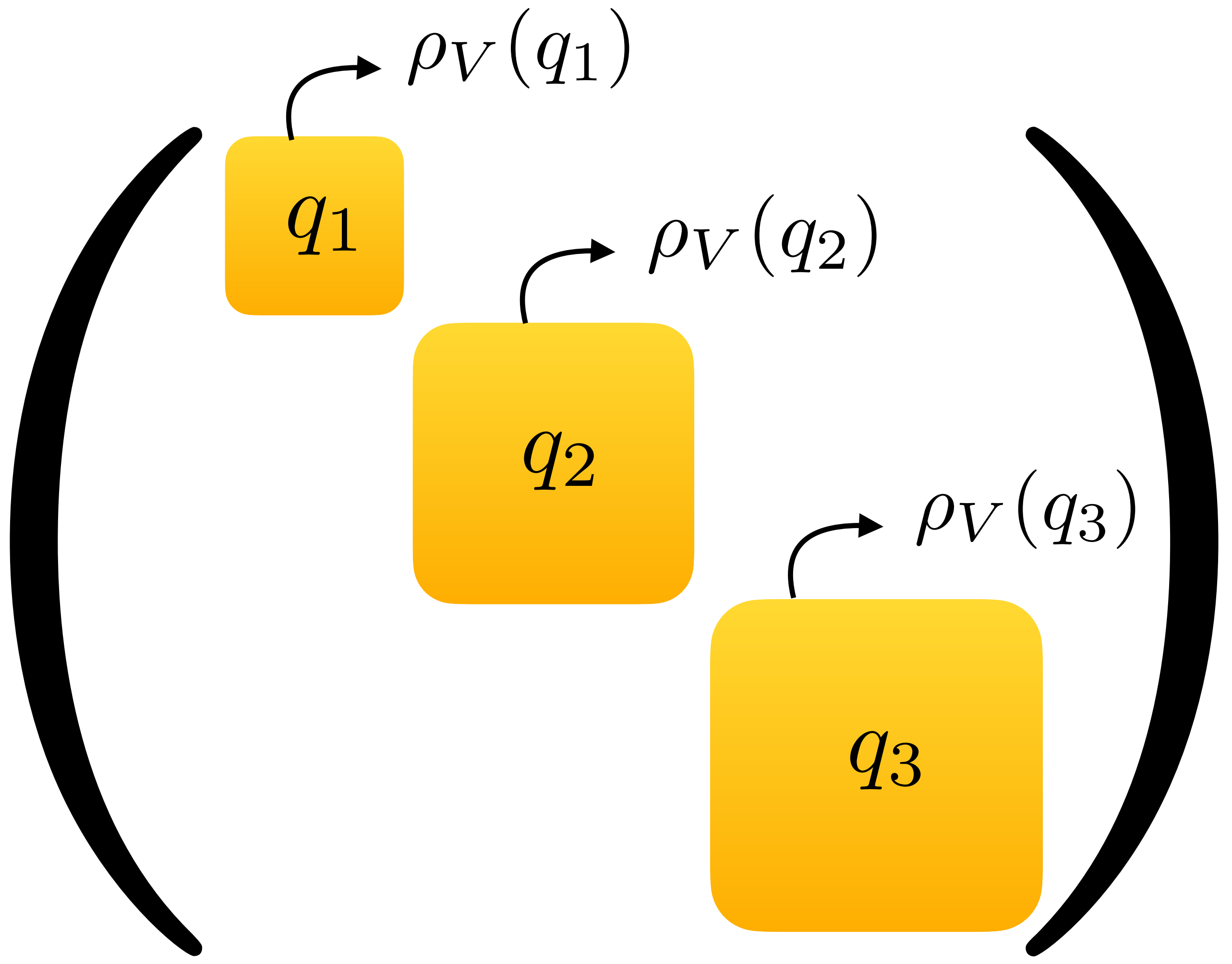
New ingredient for analyzing the fine structure of entanglement in theories endowed with a global symmetry

System with a conserved $U(1)$ charge $Q = Q_V + Q_{V'}$

$|\psi\rangle \in \mathcal{H}$ eigenvector of Q

$$[Q_V, \rho_V] = 0$$

$$\rho_V =$$



$$\rho_V(q) \equiv \frac{\Pi_V(q)\rho_V\Pi_V(q)}{\mathrm{Tr}(\Pi_V(q)\rho_V)} \longrightarrow p_V(q)$$

Symmetry-resolved entanglement entropy [Laflorencie, Rachel, ’14, Sela, Goldstein,’17]

$$S_V(q) = -\mathrm{Tr}\rho_V(q)\ln\rho_V(q) \longrightarrow S_V = \sum_q p_V(q) S_V(q) - \sum_q p_V(q)\ln p_V(q)$$

Charge sector decomposition of entanglement entropy

Equipartition of entanglement: the symmetry-resolved entanglement entropy is independent of the charge sector at leading order in the ultraviolet cutoff [Xavier, Alcaraz, Sierra, ’18]

Goal of this talk

How do the charge sectors of a theory with U(1) symmetry contribute to the modular flow?

Modular theory in different charge sectors of the theory

Example: simple field theoretical setup

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V $\longrightarrow$ vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V ⟶ vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V ⟶ vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Vacuum state of the QFT $|\psi\rangle \in \mathcal{H}$

Modular flow [Tomita, ’67; Takesaki, ’70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V ⟶ vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Vacuum state of the QFT $|\psi\rangle \in \mathcal{H}$

Δ_ψ **Modular operator**

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V → vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Vacuum state of the QFT $|\psi\rangle \in \mathcal{H}$

Δ_ψ **Modular operator**

Given $A \in \mathcal{A}(V),\, t \in \mathbb{R}$

$\Delta_\psi^{\mathrm{i}t} A \Delta_\psi^{-\mathrm{i}t}$ **Modular flow**

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V → vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

Vacuum state of the QFT $|\psi\rangle \in \mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Δ_ψ **Modular operator**

Given $A \in \mathcal{A}(V)$, $t \in \mathbb{R}$

$\Delta_\psi^{\mathrm{it}} A \Delta_\psi^{-\mathrm{it}}$ **Modular flow**

We assume $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ →

$$\Delta_\psi = \rho_V \otimes \rho_{V'}^{-1}$$

$$\Delta_\psi^{\mathrm{it}} A \Delta_\psi^{-\mathrm{it}} = \rho_V^{\mathrm{it}} A \rho_V^{-\mathrm{it}} \otimes \mathbf{1}_{V'}$$

Modular flow [Tomita, '67; Takesaki, '70]

In the algebraic approach to QFT, von Neumann (vN) algebras of operators play a special role

Causally complete region of spacetime V → vN algebra $\mathcal{A}(V) \subset$ bounded operators on $\mathcal{H}$

QFT with U(1) global symmetry: we choose $\mathcal{A}(V)$ only made of U(1) invariant operators

Vacuum state of the QFT $|\psi\rangle \in \mathcal{H}$

Δ_ψ **Modular operator**

Given $A \in \mathcal{A}(V)$, $t \in \mathbb{R}$

$\Delta_\psi^{\mathrm{i}t} A \Delta_\psi^{-\mathrm{i}t}$ **Modular flow**

We assume $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{V'}$ →

$$\Delta_\psi = \rho_V \otimes \rho_{V'}^{-1}$$

$$\Delta_\psi^{\mathrm{i}t} A \Delta_\psi^{-\mathrm{i}t} = \rho_V^{\mathrm{i}t} A \rho_V^{-\mathrm{i}t} \otimes \mathbf{1}_{V'}$$

Given $A, B \in \mathcal{A}(V)$, $t \in \mathbb{R}$

$$G_{\mathrm{mod}}(t) \equiv \langle\psi| B \Delta_\psi^{\mathrm{i}t} A \Delta_\psi^{-\mathrm{i}t} |\psi\rangle = \langle\psi| B \rho_V^{\mathrm{i}t} A \rho_V^{-\mathrm{i}t} |\psi\rangle$$

Modular correlation function

Modular flow in different charge sectors

Local operator algebra

$$\mathcal{A}(V)$$

QFT state

$$|\psi\rangle$$

Modular flow in different charge sectors

Local operator algebra

$$\mathcal{A}(V)$$

QFT state

$$|\psi\rangle$$

$$\mathcal{A}_q(V) = \Pi_V(q)\mathcal{A}(V)\Pi_V(q)$$

QFT with U(1) global symmetry: the algebra of U(1) invariant operators can be decomposed

$$\mathcal{A}(V) = \bigoplus_{q\in\mathbb{Z}} \mathcal{A}_q(V)$$

Modular flow in different charge sectors

Local operator algebra

$$\mathcal{A}(V)$$

$$\mathcal{A}_q(V) = \Pi_V(q)\mathcal{A}(V)\Pi_V(q)$$

QFT state

$$|\psi\rangle$$

$$|\psi_q\rangle = \Pi_V(q) \otimes \mathbf{1}_{V'}|\psi\rangle$$

QFT with U(1) global symmetry: the algebra of U(1) invariant operators can be decomposed

$$\mathcal{A}(V) = \bigoplus_{q\in\mathbb{Z}} \mathcal{A}_q(V)$$

Modular flow in different charge sectors

Local operator algebra

$$\mathcal{A}(V)$$

$$\mathcal{A}_q(V) = \Pi_V(q)\mathcal{A}(V)\Pi_V(q)$$

QFT state

$$|\psi\rangle$$

$$|\psi_q\rangle = \Pi_V(q) \otimes \mathbf{1}_{V'}|\psi\rangle$$

QFT with U(1) global symmetry: the algebra of U(1) invariant operators can be decomposed

$$\mathcal{A}(V) = \bigoplus_{q\in\mathbb{Z}} \mathcal{A}_q(V)$$

For each charge sector a modular operator can be defined

Given $A_q \in \mathcal{A}_q(V)$

$$\sigma_{t,q}(A_q) = [\rho_V(q)]^{\mathrm{i}t} A_q [\rho_V(q)]^{-\mathrm{i}t}$$

Modular flow in each sector

$$\sum_{q\in\mathbb{Z}} \sigma_{t,q}(A_q) = \sigma_t(A)$$

Modular flow in different charge sectors

Local operator algebra

$$\mathcal{A}(V)$$

$$\mathcal{A}_q(V) = \Pi_V(q)\mathcal{A}(V)\Pi_V(q)$$

QFT state

$$|\psi\rangle$$

$$|\psi_q\rangle = \Pi_V(q) \otimes \mathbf{1}_{V'}|\psi\rangle$$

QFT with U(1) global symmetry: the algebra of U(1) invariant operators can be decomposed

$$\mathcal{A}(V) = \bigoplus_{q\in\mathbb{Z}} \mathcal{A}_q(V)$$

For each charge sector a modular operator can be defined

Given $A_q \in \mathcal{A}_q(V)$

$$\sigma_{t,q}(A_q) = [\rho_V(q)]^{\mathrm{i}t} A_q [\rho_V(q)]^{-\mathrm{i}t}$$

Modular flow in each sector

$$\sum_{q\in\mathbb{Z}} \sigma_{t,q}(A_q) = \sigma_t(A)$$

$$G_{\mathrm{mod}}(t,q) \equiv \frac{\langle\psi_q|B_q\, \sigma_{t,q}(A_q) \otimes \mathbf{1}_{V'}|\psi_q\rangle}{\langle\psi_q|\psi_q\rangle}$$

Symmetry-resolved modular correlation function

$$G_{\mathrm{mod}}(t) = \sum_{q\in\mathbb{Z}} p_V(q) G_{\mathrm{mod}}(t,q)$$

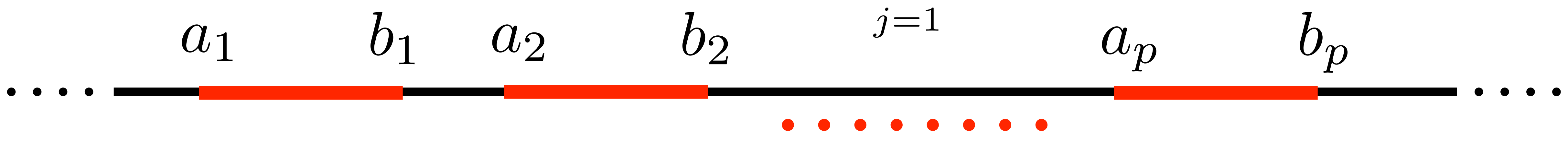
Free massless Dirac field theory in 1+1 dimensions

Symmetry resolution for $U(1)$-invariant operator: **charge density** $j(x)$

[Hollands, '21; Mintchev, Tonni, '21]

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) \equiv \langle \psi | j(x) \rho_V^{\mathrm{i}t} j(y) \rho_V^{-\mathrm{i}t} | \psi \rangle$$

$$V = \bigcup_{j=1}^{p} [a_j, b_j]$$



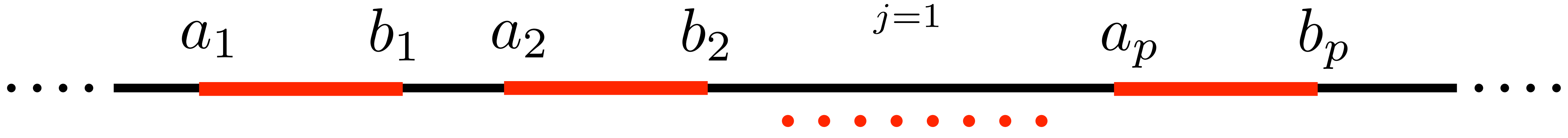
Free massless Dirac field theory in 1+1 dimensions

Symmetry resolution for $U(1)$-invariant operator: **charge density** $j(x)$

[Hollands, '21; Mintchev, Tonni, '21]

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) \equiv \langle \psi | j(x) \rho_V^{\mathrm{i}t} j(y) \rho_V^{-\mathrm{i}t} | \psi \rangle$$

$$V = \bigcup_{j=1}^{p} [a_j, b_j]$$



Symmetry resolution

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) + \frac{q}{\ln \epsilon}\mathcal{G}(x,y;t) + \dots$$

terms vanishing as $\epsilon \to 0$

$$\sum_{q \in \mathbb{Z}} p_V(q) G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t)$$

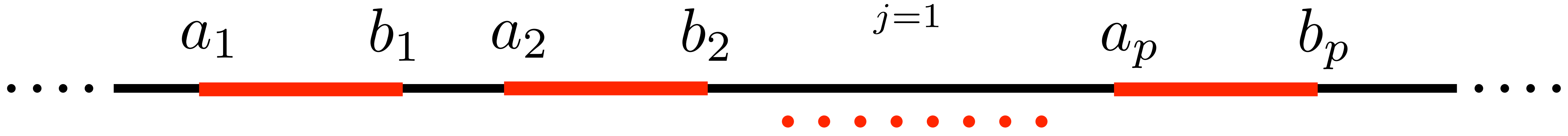
Free massless Dirac field theory in 1+1 dimensions

Symmetry resolution for $U(1)$-invariant operator: **charge density** $j(x)$

[Hollands, '21; Mintchev, Tonni, '21]

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) \equiv \langle \psi | j(x) \rho_V^{\mathrm{i}t} j(y) \rho_V^{-\mathrm{i}t} | \psi \rangle$$

$$V = \bigcup_{j=1}^{p} [a_j, b_j]$$



Symmetry resolution

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) + \frac{q}{\ln \epsilon}\mathcal{G}(x,y;t) + \dots$$

terms vanishing as $\epsilon \to 0$

$$\sum_{q\in\mathbb{Z}} p_V(q) G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t)$$

- **SR modular correlation function is well-defined as** $\epsilon \to 0$

Its definition arguably works independently of the factorization of the Hilbert space

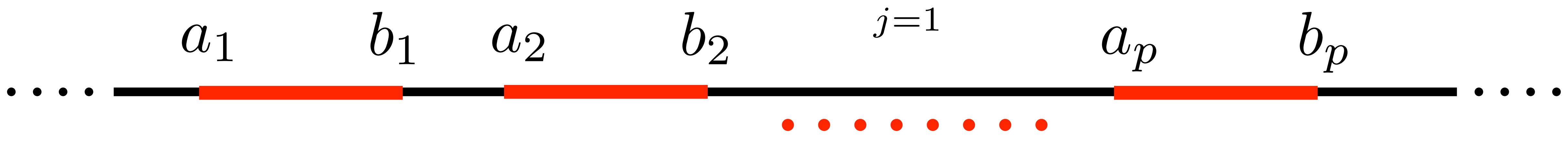
Free massless Dirac field theory in 1+1 dimensions

Symmetry resolution for $U(1)$-invariant operator: **charge density** $j(x)$

[Hollands, '21; Mintchev, Tonni, '21]

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) \equiv \langle\psi|j(x)\rho_V^{\mathrm{i}t} j(y)\rho_V^{-\mathrm{i}t}|\psi\rangle$$

$$V = \bigcup_{j=1}^{p}[a_j, b_j]$$



Symmetry resolution

$$G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t) + \frac{q}{\ln\epsilon}\mathcal{G}(x,y;t) + \dots$$

terms vanishing as $\epsilon \to 0$

$$\sum_{q\in\mathbb{Z}} p_V(q) G^{\mathrm{D}}_{\mathrm{mod}}(x,y;q,t) = G^{\mathrm{D}}_{\mathrm{mod}}(x,y;t)$$

- **SR modular correlation function is well-defined as** $\epsilon \to 0$

Its definition arguably works independently of the factorization of the Hilbert space

- Independence of the charge sector as $\epsilon \to 0$ ⟶ **Equipartition of the modular correlation function**

Conclusions and outlook

Consistent definition of modular flow in each charge sector

Symmetry-resolved modular correlation function of the charge density operator in 1+1-dimensional massless Dirac theory

Equipartition of the modular correlation function

Conclusions and outlook

Consistent definition of modular flow in each charge sector

Symmetry-resolved modular correlation function of the charge density operator in 1+1-dimensional massless Dirac theory

Equipartition of the modular correlation function

Future perspectives

General construction without Hilbert space factorization

Does the equipartition hold also for all the U(1)-invariant operators of the theory?

Connection with equipartition of entanglement

Analysis for other theories and more complicated symmetry groups?

Thank you for your attention!